Algebra Cheat Sheet

Basic Properties & Facts

Arithmetic Operations

\[ ab + ac = a(b + c) \]
\[ \frac{a}{b} = \frac{a}{b} \]
\[ a + c = b + c \]
\[ a - c = b - c \]
\[ \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \]
\[ a - \frac{c}{d} = \frac{ad - bc}{bd} \]
\[ \frac{a - b}{c - d} = \frac{b - a}{d - c} \]
\[ \frac{ab + ac}{a} = b + c, \ a \neq 0 \]

Exponent Properties

\[ a^n a^m = a^{n+m} \]
\[ \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}} \]
\[ (a^n)^m = a^{mn} \]
\[ a^0 = 1, \ a \neq 0 \]
\[ (ab)^n = a^n b^n \]
\[ a^{-n} = \frac{1}{a^n} = \frac{1}{a^{n}} = a^n \]
\[ \left( \frac{a}{b} \right)^n = \left( \frac{b}{a} \right)^n = \frac{b^n}{a^n} \]
\[ a^n = \left( a^\frac{1}{n} \right)^n \]

Properties of Radicals

\[ \sqrt[n]{a^m} = a^{\frac{m}{n}} \]
\[ \sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab} \]
\[ \sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{a} \]
\[ \sqrt[n]{\sqrt[n]{b}} = \sqrt[n]{b} \]
\[ \sqrt[n]{a^n} = a, \text{ if } n \text{ is odd} \]
\[ \sqrt[n]{a^n} = |a|, \text{ if } n \text{ is even} \]

Properties of Inequalities

If \( a < b \) then \( a + c < b + c \) and \( a - c < b - c \)
\[
\begin{align*}
\text{If } a < b \text{ and } c > 0 \text{ then } ac < bc \text{ and } \frac{a}{c} < \frac{b}{c} \\
\text{If } a < b \text{ and } c < 0 \text{ then } ac > bc \text{ and } \frac{a}{c} > \frac{b}{c}
\end{align*}
\]

Properties of Absolute Value

\[ |a| = \begin{cases} 
 a & \text{if } a \geq 0 \\
 -a & \text{if } a < 0 
\end{cases} \]
\[ |a| \geq 0 \]
\[ |a| = |b| \]
\[ |a + b| \leq |a| + |b| \text{ Triangle Inequality} \]

Distance Formula

If \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \) are two points the distance between them is
\[
d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Complex Numbers

\[ i = \sqrt{-1} \]
\[ i^2 = -1 \]
\[ \sqrt{-a} = i\sqrt{a}, \ a \geq 0 \]
\[ (a + bi) + (c + di) = a + c + (b + d)i \]
\[ (a + bi) - (c + di) = a - c + (b - d)i \]
\[ (a + bi)(c + di) = ac - bd + (ad + bc)i \]
\[ (a + bi)(a - bi) = a^2 + b^2 \]
\[ |a + bi| = \sqrt{a^2 + b^2} \text{ Complex Modulus} \]
\[ \overline{(a + bi)} = a - bi \text{ Complex Conjugate} \]
\[ (a + bi)(a + bi) = |a + bi|^2 \]
Logarithms and Log Properties

Definition

\[ y = \log_b x \] is equivalent to \[ x = b^y \]

Example

\[ \log_5 125 = 3 \] because \[ 5^3 = 125 \]

Special Logarithms

\[ \ln \log x \] natural log

\[ \log \log x \] common log

\[ e^x = x \]

where \( e \approx 2.718281828 \)

Logarithm Properties

\[ \log_b b^y = y \]

\[ \log_b 1 = 0 \]

\[ \log_b b^x = x \]

\[ \log_b (x^r) = r \log_b x \]

\[ \log_b (xy) = \log_b x + \log_b y \]

\[ \frac{\log_b x}{\log_b y} = \log_b \frac{x}{y} = \log_b x - \log_b y \]

The domain of \( \log_b x \) is \( x > 0 \)

Factoring and Solving

Factoring Formulas

\[ x^2 - a^2 = (x + a)(x - a) \]

\[ x^2 + 2ax + a^2 = (x + a)^2 \]

\[ x^2 - 2ax + a^2 = (x - a)^2 \]

\[ x^2 + (a + b)x + ab = (x + a)(x + b) \]

\[ x^3 + 3ax^2 + 3a^2 x + a^3 = (x + a)^3 \]

\[ x^3 - 3ax^2 + 3a^2 x - a^3 = (x - a)^3 \]

\[ x^3 + a^3 = (x + a)(x^2 - ax + a^2) \]

\[ x^3 - a^3 = (x - a)(x^2 + ax + a^2) \]

\[ x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n) \]

If \( n \) is odd then,

\[ x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + \ldots + a^{n-1}) \]

\[ x^n + a^n = (x + a)(x^{n-1} - ax^{n-2} + \ldots - a^{n-1}) \]

Quadratic Formula

Solve \( ax^2 + bx + c = 0, \ a \neq 0 \)

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

If \( b^2 - 4ac > 0 \) - Two real unequal solns.

If \( b^2 - 4ac = 0 \) - Repeated real solution.

If \( b^2 - 4ac < 0 \) - Two complex solutions.

Square Root Property

If \( x^2 = p \) then \( x = \pm \sqrt{p} \)

Absolute Value Equations/Inequalities

If \( b \) is a positive number

\[ |p| = b \implies p = -b \text{ or } p = b \]

\[ |p| < b \implies -b < p < b \]

\[ |p| > b \implies p < -b \text{ or } p > b \]

Completing the Square

Solve \( 2x^2 - 6x - 10 = 0 \)

(1) Divide by the coefficient of the \( x^2 \)

\[ x^2 - 3x - 5 = 0 \]

(2) Move the constant to the other side.

\[ x^2 - 3x = 5 \]

(3) Take half the coefficient of \( x \), square it and add it to both sides

\[ x^2 - 3x + \left(\frac{-3}{2}\right)^2 = 5 + \left(\frac{-3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4} \]

(4) Factor the left side

\[ \left(x - \frac{3}{2}\right)^2 = \frac{29}{4} \]

(5) Use Square Root Property

\[ x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2} \]

(6) Solve for \( x \)

\[ x = \frac{3}{2} \pm \frac{\sqrt{29}}{2} \]
Functions and Graphs

Constant Function
\[ y = a \quad \text{or} \quad f(x) = a \]
Graph is a horizontal line passing through the point \((0, a)\).

Line/Linear Function
\[ y = mx + b \quad \text{or} \quad f(x) = mx + b \]
Graph is a line with point \((0, b)\) and slope \(m\).

Slope
Slope of the line containing the two points \((x_1, y_1)\) and \((x_2, y_2)\) is
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} \]

Slope – intercept form
The equation of the line with slope \(m\) and y-intercept \((0, b)\) is
\[ y = mx + b \]

Point – slope form
The equation of the line with slope \(m\) and passing through the point \((x_1, y_1)\) is
\[ y = y_1 + m(x - x_1) \]

Parabola/Quadratic Function
\[ y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k \]
The graph is a parabola that opens up if \(a > 0\) or down if \(a < 0\) and has a vertex at \((h, k)\).

Parabola/Quadratic Function
\[ y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c \]
The graph is a parabola that opens up if \(a > 0\) or down if \(a < 0\) and has a vertex at \(\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)\).

Parabola/Quadratic Function
\[ x = ay^2 + by + c \quad g(y) = ay^2 + by + c \]
The graph is a parabola that opens right if \(a > 0\) or left if \(a < 0\) and has a vertex at \(\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)\).

Circle
\[ (x - h)^2 + (y - k)^2 = r^2 \]
Graph is a circle with radius \(r\) and center \((h, k)\).

Ellipse
\[ \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \]
Graph is an ellipse with center \((h, k)\) with vertices \(a\) units right/left from the center and vertices \(b\) units up/down from the center.

Hyperbola
\[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \]
Graph is a hyperbola that opens left and right, has a center at \((h, k)\), vertices \(a\) units left/right of center and asymptotes that pass through center with slope \(\pm \frac{b}{a}\).

Hyperbola
\[ \frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1 \]
Graph is a hyperbola that opens up and down, has a center at \((h, k)\), vertices \(b\) units up/down from the center and asymptotes that pass through center with slope \(\pm \frac{b}{a}\).
### Common Algebraic Errors

<table>
<thead>
<tr>
<th>Error</th>
<th>Reason/Correct/Justification/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$</td>
<td>Division by zero is undefined!</td>
</tr>
<tr>
<td>$-3^2 \neq 9$</td>
<td>$-3^2 = -9, \ (\ -3\ )^2 = 9$ Watch parenthesis!</td>
</tr>
<tr>
<td>$(x^2)^3 \neq x^3$</td>
<td>$(x^2)^3 = x^6$</td>
</tr>
<tr>
<td>$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$</td>
<td>$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$</td>
</tr>
<tr>
<td>$\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$</td>
<td>A more complex version of the previous error.</td>
</tr>
<tr>
<td>$a + bx \neq 1 + bx$</td>
<td>Beware of incorrect canceling!</td>
</tr>
<tr>
<td>$-a(x-1) \neq -ax - a$</td>
<td>$-a(x-1) = -ax + a$</td>
</tr>
<tr>
<td>$(x+a)^2 \neq x^2 + a^2$</td>
<td>$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$</td>
</tr>
<tr>
<td>$\sqrt{x^2 + a^2} \neq x + a$</td>
<td>$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$</td>
</tr>
<tr>
<td>$\sqrt{x + a} \neq \sqrt{x} + \sqrt{a}$</td>
<td>See previous error.</td>
</tr>
<tr>
<td>$(x+a)^n \neq x^n + a^n$ and $\sqrt{x + a} \neq \sqrt{x} + \sqrt{a}$</td>
<td>More general versions of previous three errors.</td>
</tr>
<tr>
<td>$2(x+1)^2 \neq (2x+2)^2$</td>
<td>$2(x+1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$</td>
</tr>
<tr>
<td>$(2x+2)^2 \neq 2(x+1)^2$</td>
<td>Square first then distribute!</td>
</tr>
<tr>
<td>$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 + a^2}$</td>
<td>See the previous example. You can not factor out a constant if there is a power on the parenthesis!</td>
</tr>
</tbody>
</table>

For a complete set of online Algebra notes visit [http://tutorial.math.lamar.edu](http://tutorial.math.lamar.edu). © 2005 Paul Dawkins